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Results of a calculation and experimental study of current leads are presented.

Current leads in superconductive cables, in contrast to submersible ones, are characterized by independence of cooling agent flow rate upon heat influx into the cold zone. The problem of optimizing such current leads was solved in [1-3]. The goal of the present study is to obtain the conditions for sufficient heat exchange in the current lead, to calculate the lead construction with consideration of variability in the properties of helium, and experimental study of a current lead of porous construction.

We will write the thermal balance equations for a current lead, taking the lead temperature  $T$  as the independent variable [4], in dimensionless form

$$\frac{dt}{dT} = - \frac{\Gamma \Lambda (T - t)}{C_p G Q}, \tag{1}$$

$$\frac{dQ}{dT} = - \frac{T}{Q} \text{Ln} + \frac{\Gamma \Lambda (T - t)}{Q} \tag{2}$$

with boundary conditions

$$t|_{T=T_0} = T_0, \quad Q|_{T=T_L} = 0, \tag{3}$$

where

$$Q = Q^*/t_0^* \text{Ln}^{*1/2}; \quad t = t^*/t_0^*; \quad T = T^*/t_0^*; \quad G = G^*C_p^0/i^* \text{Ln}^{*1/2};$$

$$C_p = C_p^*/C_p^0; \quad \Gamma = \Gamma^* \Lambda^0 S^*/\text{Ln}^* i^{*2}; \quad \Lambda = \Lambda^*/\Lambda^0; \quad \text{Ln} = \kappa^* \Lambda^*/T^* \text{Ln}^*;$$

$$\Lambda_1 = \Lambda^0 S^*/L^* i^* \text{Ln}^{*1/2}.$$

We will seek the optimum flow rate, that is, the flow rate providing minimum normalized energy expenditure, which was defined in [5] in the form

$$\dot{E}^* = \dot{G}^* C_p^* (t_{sm}^* \ln \frac{t_L^*}{t_0^*} - t_L^* + t_0^*) - \frac{t_{sm}^* - t_0^*}{t_0^*} \dot{Q}_0^*. \tag{4}$$

The nonlinear boundary problem (1)-(3) was solved by Newton's linearization method with iterations. We write

$$t^{k+1} = t^k + \delta t, \quad Q^{k+1} = Q^k + \delta Q. \tag{5}$$

We then substitute Eq. (5) in system (1)-(2), and, omitting the subscript  $k$  on  $Q$  and  $t$ , we obtain

$$\frac{d(\delta t)}{dT} - \frac{\Gamma \Lambda}{G Q C_p} \delta t - \frac{\Gamma \Lambda (T - t)}{G Q^2 C_p} \delta Q = - \frac{dt}{dT} - \frac{\Gamma \Lambda (T - t)}{G Q C_p}, \tag{6}$$

$$\frac{d(\delta Q)}{dT} - \left[ \frac{T}{Q^2} \text{Ln} - \frac{\Gamma \Lambda}{Q^2} (T - t) \right] \delta Q + \frac{\Gamma \Lambda}{Q} \delta t = - \frac{dQ}{dT} - \frac{T}{Q} \text{Ln} + \frac{\Gamma \Lambda}{Q} (T - t). \tag{7}$$

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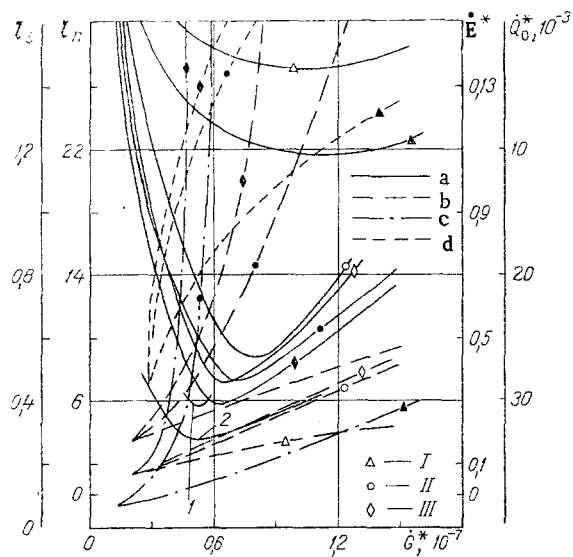


Fig. 1. Results of calculation of current guides made of copper  $M_1$  (shaded points) and  $M_3$  (open points), shunted by a superconductor: 1) ideal heat exchange with composite current lead of [1]; 2) in pure copper current lead; a) energy losses; b) length of normal section; c) length of superconductive section of current lead; d) thermal flux into cold zone; I)  $\Gamma_\lambda = 1$ ; II) 3; III) 5.  $E^*$ ,  $Q_0^*$ , W/A.

The system of linear equations (6)-(7) was approximated by finite-difference equations on a nonuniform grid [6], which were solved by the matrix drive method, after which iteration was performed for the nonlinearity.

The geometric size parameter  $\Lambda_1$  is determined by substitution of the solution of system (1)-(2) with conditions (3) in the Fourier thermal conductivity law, which gives

$$\Lambda_1^{-1} = - \int_{T_0}^{T_L} \frac{\Lambda}{Q} dT. \quad (8)$$

With knowledge of  $\Lambda_1$  and specification of the lead cross section the optimum length can be determined.

The quality of current lead cooling is determined by the product  $\Gamma_\lambda = \Gamma \cdot \Lambda$ . The value of this parameter varies along current lead length due to the variability of  $\Gamma$  and  $\Lambda$ .

A number of studies have taken the heat-exchange parameter  $\Gamma$  to be constant, but in practice it is not constant, since the thermal conductivity coefficient of helium is temperature dependent. Expressing  $\Gamma$  on the basis of the criterial dependence of heat exchange in forced convective flow:

$$Nu = A Re^n Pr^m, \quad (9)$$

we obtain

$$\Gamma = B \left( \frac{\lambda^0}{\lambda^*} \right) G^n, \quad (10)$$

where

$$B = A Pr^{m-n} \frac{\lambda^0}{d_c^*} \frac{P^* \lambda^0 S^*}{\dot{I}^{*2} L n^*} \left( \frac{d_c^{*1/2} L n^{*1/2}}{\lambda^0 S_c^*} \right)^n,$$

and  $d_c^*$  and  $S_c^*$  are the equivalent diameter and cross section of the helium channels;  $\lambda^0$  is the scale factor for the helium thermal conductivity coefficient.

Under thermodynamic conditions far from critical the Prandtl number depends weakly on temperature. Neglecting this dependence, we find that the coefficient  $B$  is not related to temperature or flow rate, and is defined only by the construction of the current lead. According to [10], the parameter  $\Gamma$  depends significantly on the exponent  $n$ , which for laminar completely steady-state flow is equal to zero, equal to 0.8 for turbulent flow, and close to unity for flow in a porous body [7].

Three series of calculations were performed, in each of which one of the parameters  $\Gamma$ ,  $\Gamma_\lambda$ , or  $B$  was assumed constant along the length of the lead. The parameter values were varied together with the flow rate.

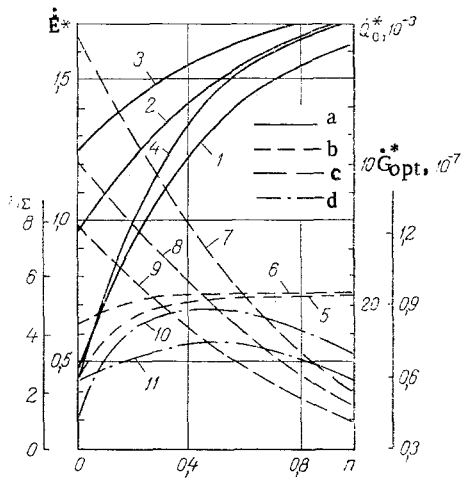


Fig. 2. Thermal flux (a), current lead length at optimum flow rate (b), minimum energy losses (c), and optimum flow rate (d) vs. exponent  $n$ : 1)  $B = 3$ ; 2) 5; 3) 9; 4) 3; 5) 3; 6) 9; 7) 3; 8) 6; 9) 9; 10) 3; 11) 9; 1-3, 5-11)  $M_3$  copper; 4)  $M_1$ ;  $\dot{E}^*$ , W/A;  $\dot{Q}_0^*$ , W/A;  $\dot{G}_{opt}^*$ , kg/(sec·A).

Figure 1 shows results of a calculation for the condition  $\partial \Gamma_\lambda / \partial T = 0$  for copper current guides  $M_1$  and  $M_3$  with optimum geometric dimensions, shunted by a superconductor with critical temperature of  $16^\circ\text{K}$ . The properties of the  $M_3$  copper were taken from [2], those of  $M_1$  from [8].

The minimum in the curve of energy loss vs. flow rate becomes deeper with increase in  $\Gamma_\lambda$ . The value  $\Gamma_\lambda = 5$  can be considered the lower limit of heat exchange, approaching ideal. Minimum energy losses are then close to the energy losses of an ideally cooled lead, and thermal fluxes into the cold zone are completely removed at a flow rate approximately 30% higher than the optimum flow rate. It is advantageous to increase heat exchange to the value  $\Gamma_\lambda = 5$ , with further increase in  $\Gamma_\lambda$  giving no significant improvement in energy loss, only degrading hydraulic characteristics. This condition is significant for the "cold" end of the current lead, and agrees with the results presented in [3]. Figure 1 also shows dimensionless optimum lengths of the superconductive and normally conducting sections.

Calculations for the condition  $\Gamma = \text{const}$  [6] show that to reach a heat exchange close to ideal it is necessary that  $\Gamma$  be of the order of magnitude of 20 or higher, i.e., heat exchange in the "cold" section determines the cooling characteristics of the current guide.

In the series of calculations for  $B = \text{const}$  the value of the exponent in the function  $\text{Nu} = f(\text{Re})$  was varied. Results are shown in Fig. 2.

With increase in  $n$  the value of  $B$  corresponding to zero flux into the cold zone decreases.

As is evident from Fig. 2, the length of the normally conductive section of the current lead for  $n > 0.4$  is practically independent of  $n$ , while with increase in  $B$  and  $n$  the length of the superconductive section increases rapidly, and the flow rates corresponding to minimum energy expenditure shift toward higher values as compared to those for the case  $\Gamma = \text{const}$  and  $\Gamma_\lambda = \text{const}$ , due to the dependence of  $\Gamma$  on flow rate. And, in general, the dependence of  $G_{opt}$  on the material of the current-carrying section and cooling conditions is relatively weak. In the majority of calculations performed for various types of copper, aluminum, and brass for various fixed  $\Gamma$ ,  $\Gamma_\lambda$ ,  $B$ , the optimum flow rate can be defined as  $2.5 < G_{opt} < 4$ . At a heat exchange close to ideal it follows from the calculations that the optimum flow rate proves equal to the intrinsic flow rate of an optimal submerged current lead. The same follows from Eq. (4), at  $t_L^* = t_{sm}^*$  and  $\dot{G}^* = r^* \dot{Q}_0^*$ , where  $r^*$  is the heat of phase transition.

Positive results are obtained by shunting the current lead with a superconductor only for materials with a high thermal conductivity coefficient at helium temperature and for good cooling. In the opposite case the segment of the current lead across which the temperature drop ( $T_0$ ,  $T_S$ ) is concentrated proves to be quite short, and heat removal therefore is insignificant.

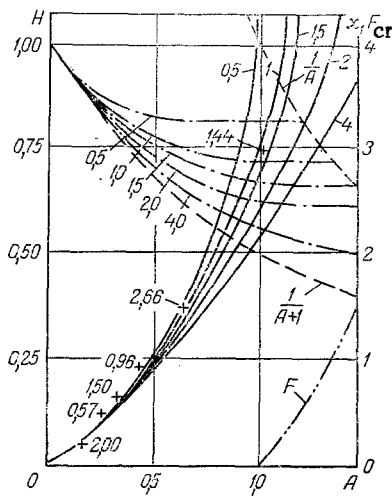


Fig. 3

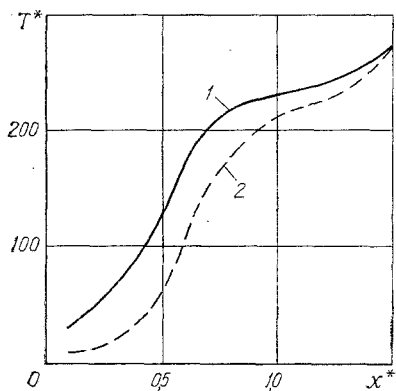


Fig. 4

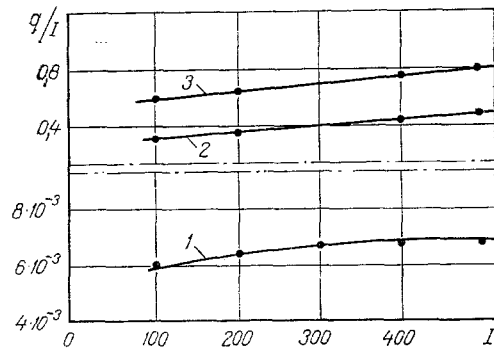


Fig. 5

Fig. 3. Results of analytical calculation with Eq. (11).

Fig. 4. Temperature distribution in current lead: 1) calculated curve; 2) experiment.  $T^*$ , K;  $x^*$ , m.

Fig. 5. Thermal influx vs. current for current leads: 1) brass tube with porous bronze insert; 2) copper tube with porous copper insert; 3) copper tube.  $q/I$ , MW/A;  $I$ , A.

To improve heat exchange at the "cold" end and make it possible to transfer current into the superconductor, it is necessary to know the length of the superconductive segment. The latter can be determined from the calculations presented above, and also from an analytical expression obtained in the following manner: we assume the thermophysical properties in the interval  $(T_0, T_S)$  constant, equal to their mean values, taking  $l_n = 0$  in Eq. (2), reduce Eqs. (1)-(2), to a quadrature, integrate, and find

$$\frac{2GC_p x_S}{\Lambda \lambda_1} = \frac{F}{D_1} \ln \left| \frac{1 + F - D_1}{1 + F + D_1} \frac{1 + H_S F - D_1}{1 + H_S F + D_1} \right| = x_1, \quad (11)$$

where

$$D_1 = \sqrt{1 + 2F}; \quad F = \frac{2G^2}{\Gamma \Lambda}; \quad H_S = \frac{-Q_S}{-Q_0 + G(T_S - T_0)},$$

and  $H_S$  is defined by the transcendental equation

$$(1 - AH_S)f(H_S, F) = f(1, F),$$

where

$$f(H, F) = (F^2 H^2 + 2HF - 2F)^{-1/2} \left| \frac{1 + HF - D_1}{1 + HF + D_1} \right|^{D_1/2},$$

$$A = G(T_S - T_0)/(-Q_S).$$

The functions  $H_S$  and  $x_1(A)$  for various  $F$  are shown in Fig. 3. The crosses denote  $x_1$  values obtained from the numerical calculation described previously. Shown with them are corresponding values of  $F$ .

The  $H_S(A)$  curves lie between the limiting curves  $H_{S1} = 1/(1 + A)$  and  $H_{S2} = 1/A$ , which correspond to the absence of heat exchange and ideal heat exchange.

In the case of ideal heat exchange ( $H_S \rightarrow 1/A$ ,  $F \rightarrow 0$ ),

$$x_S^* = \frac{\lambda^* S^*}{C_p^* G^*} \ln \left( \frac{1}{1 - A} \right)$$

the condition for existence of a solution is  $A < 1$ . For  $F \neq 0$ , such a result can be obtained from analysis of Eq. (11). A solution with monotonic temperature distribution is impossible if

$$\frac{D_1 - 1}{F} > \frac{1}{A}. \quad (12)$$

The function  $F_{Cr}(A)$  following from equality of both parts of Eq. (12) is shown in Fig. 3. For  $A > 1$  and  $F \rightarrow F_{Cr}(A)$ , complete heat removal is accomplished in the superconductive section. For a fixed flow rate  $G$ , we can use  $F_{Cr}$  to determine the heat-exchange intensity parameter  $\Gamma A$  at which zero heat influx into the cold zone is achieved.

Specifying the values  $G = 2.5$ ,  $Q_S^*/\dot{I}^* = 2$  mW/A,  $T_S = T_0 = 11^\circ\text{K}$ , then  $F_{Cr} = 2.15$  and  $\Gamma A \approx 6$ .

Thus, from analysis of heat exchange in the superconductive section we have the necessary condition for creation of a heat-exchange intensity such as was obtained previously for the entire current lead.

Use of porous current lead construction allows achievement of a specified heat-exchange intensity due to their large specific surfaces and small volumes. Moreover, with such a cooling method  $n \approx 1$  and the decrease in thermal conductivity of helium at low temperatures has practically no effect on heat exchange. A shortcoming of such current leads is the increase in hydraulic resistance as compared to channels. The radial filtration method considered below allows elimination of this shortcoming to some extent.

Efficiency of heat exchange is insured by redistribution of cooling gas flows in the low-temperature porous section and decreases the temperature at the "cold" end, leading to minimum heat influx.

An experimental study was performed on a current lead constructed of a 1820-mm-long brass tube with outer diameter of 20 mm and wall thickness of 1 mm. A porous bronze ring-insert with superconductor was attached to its "cold" end. Its dimensions were: length 150 mm, diameter 17 mm, wall thickness 3.5 mm. The brass tube was in direct contact with the superconductor of the bronze ring-insert, so that current passed from the tube into the superconductor. The bronze matrix, having a low thermal conductivity, acted as a thermal insulation insert, reducing the thermal influx to the cable due to thermal conductivity almost to zero.

Studies were performed on the laboratory model superconductive cable described in [9].

A comparison of calculated results with experiment is shown in Fig. 4, with the example of temperature distribution over current lead length. The gas flow through the current lead was 0.5 g/sec. In the "warm" zone the calculated and experimental data coincide sufficiently well, with somewhat worse agreement in the "cold" section, which can apparently be explained by the low accuracy of the data taken from the literature for thermal conductivity, specific heat, and other characteristics of the material and their temperature dependence.

Experimental results in determination of thermal influx to the "cold" zone are shown in Fig. 5, where for comparison data are shown for two current leads made of copper tube with

and without a porous insert. Tube dimensions are the same as the brass lead described above. Increase in helium flow rate from 0.15 to 1.5 g/sec has practically no effect on the thermal influx along the current lead with the porous insert, while for the lead with no insert this dependence is significant. The data of Fig. 5 correspond to a helium flow rate of 0.5 g/sec and a load current varying from 100 to 500 A. The thermal influx is two orders of magnitude smaller than for the lead without the insert. Since the porous insert makes it possible to create both axial and radial helium flow, it is possible to regulate the value of the thermal influx by redistributing the flow in either the radial or the axial direction with the same helium flow at the input to the current lead [10], which is especially important for current leads designed for currents in the tens of kA. It should be noted that the condition for reliable operation of such a current lead is maintenance over the entire length of the porous insert of a working temperature lower than the critical temperature of the superconductor. In the experiments an optimum insert length of 150 mm was selected.

#### NOTATION

$T^*$ , temperature of current lead;  $t^*$ , gas temperature;  $Q^*$ , thermal flux;  $\dot{I}^*$ , current;  $\Lambda^*$ , thermal conductivity of metal;  $\lambda^*$ , thermal conductivity of gas;  $S^*$ , cross section;  $L^*$ , length of current lead;  $\Gamma^*$ , heat-exchange intensity per unit length,  $W/(m \cdot K)$ ;  $t^o = t_{sm} - t_o$ ;  $G^*$ , flow rate;  $C_p^*$ , specific heat;  $Ln^* = 2.448 \cdot 10^{-8}$ , Lorentz number,  $W^2/(A^2 \cdot K^2)$ ;  $\kappa^*$ , electrical resistivity;  $P^*$ , cooled perimeter;  $ln = Ln^{*1/2} \dot{I}^* x_n^* / \Lambda^o S^*$ ,  $l_S = Ln^{*1/2} \dot{I}^* x_S^* / \Lambda^o S^*$ ,  $l_\Sigma = l_n + l_S$ ,  $x = 2GC_p l_S / \Lambda \Lambda_1$ ,  $x_S = x_S^* / L$ . Subscripts: \*, dimensional quantity; 0, cold end of current lead; sm, at parameters of surrounding medium at warm end; S, at critical temperature of superconductor; a dot above a variable indicates a reduced value referenced to the current.

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